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Ideals and Filters on a Lattice in Neutrosophic Setting

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Abstract

The notions of ideals and filters have studied in many algebraic (crisp) fuzzy structures and used to study their various properties, representations and characterizations. In addition to their theoretical roles, they have used in some areas of applied mathematics. In a recent paper, Arockiarani and Antony Crispin Sweety have generalized and studied these notions with respect to the concept of neutrosophic sets introduced by Smarandache to represent imprecise, incomplete and inconsistent information. In this article, we aim to deepen the study of these important notions on a given lattice in the neutrosophic setting. We show their various properties and characterizations, in particular, we pay attention to their characterizations based on of the lattice min and max operations. In addition, we study the notion of prime single-valued neutrosophic ideal (respectively, filter) as interesting kind and we discuss some its set-operations, complement and associate sets.

Keywords: Lattice; Ideal; Filter; Single-valued neutrosophic set; SVN-lattice; SVN-ideal; SVN-filter

MSC 2010 No.: 03B52, 06B10

1. Introduction

The notions of ideals and filters are well known in many algebraic structures (e.g., semi-groups, rings, MV-algebras, lattices, etc.). They have been applied in different subjects of mathematics, see, e.g., topological spaces (Willard (1970)), metric spaces (Bourbaki (2007)) and congruence relations (Van Gasse et al. (2010)). They have been used as tools in the representations of Boolean algebra (Stone (1936)) and distributive lattice (Davey and Priestley (2002); Schröder (2002)). Also, in the theory of Lukasiewicz and Post algebras (Cignoli (1970)), as they are the kernels of the homomorphisms into the power set subalgebras. In ring theory (Mukherjee and Sen (1987)), ideals generalize certain subsets of the integers, such as the even numbers or the multiples.

Zadeh (1965) has introduced the theory of fuzzy sets (FSs), and after that several authors have conducted on the generalizations of this heory. For instance, Atanassov (1986) has introduced the theory of intuitionistic fuzzy sets (IFSs) as an extension of of FSs theory. In fuzzy setting, the falsity-membership of an element x is fixed and it can be calculated as the negation of the truth-membership of x. In the intuitionistic fuzzy setting, the falsity-membership is an independent degree satisfies the condition that is less or equal the negation of the truth-membership.

Although FSs (respectively, IFSs) are useful in handling uncertainties arising from vagueness, imprecise and incomplete information, it cannot model all sorts of indeterminate or inconsistent information that exists in real-life. Inspired by this situation, Smarandach (1998) has introduced the theory of neutrosophy to study the nature, origin and neutrality. To that end, he has introduced the theory of neutrosophic sets (NSs) as a new generalization of FSs and IFSs theories, and for the purpose to ensure the best handling of incomplete, indeterminate and inconsistent information.

The notion of NSs described by three degrees: truth-degree ($\mathcal{T}$), indeterminacy-degree ($\mathcal{I}$) and falsity-degree ($\mathcal{F}$). For the purpose of more practical use of neutrosophic sets, several authors have considered alternative conditions for the neutrosophic set degrees. The more general alternative is that considered by Wang et al. (2010), they have defined the "single-valued neutrosophic set (SVNS)" as a subclass of NSs, which can independently express the truth, the indeterminacy and the falsity degrees. These three components of a SVNS are independent and their values are enclosed in the interval [0,1]. SVNSs have considered in very significant research areas such as image processing (Guo and Cheng (2009); Guo et al. (2014)), medical diagnosis (Krohling and Campanharo (2011); Pramanik and Mondal (2015); Ye (2015)), decision making (Al-Sharqiet al. (2021); Liu and Li (2017)) and social problems (Mondal and Pramanik (2014)). More details on applications of NSs can be found in (Mary Margaret and Trinita Pricilla (2021); Bakro et al. (2021); Smarandach and Pramanik (2016)).

Due to the importance of the notions of ideals and filters in the study of several mathematical structures, as in fuzzy and intuitionistic fuzzy setting, several papers studied different notions of ideals (respectively, filters) on different extensive fuzzy structures. Kim and Jun (2001) discussed the notion of intuitionistic fuzzy ideal (IF-ideal) in a semigroup, while Banerjee and Basnet (2003) defined a similar notion on a ring structure. Recently, Akram and Dudek (2009) investigated intuitionistic fuzzy ideals on Lie algebras. In particular, Thomas and Nair (2010); (2011) introduced the notion of IF-ideal using the idea of fuzzification the membership function of elements on the carrier of a crisp lattice, and investigated some of its properties. (Boudaoud et al. (2020); Milles et al. (2017)) characterized the notions of IF-ideals and IF-filters based on of the lattice min and max operations.

Similar studies of the notions of ideals and filters in neutrosophic context have been done by several authors. For instance, Salama and Smarandache (2013) considered the notion of filters via neutrosophic crisp set and investigated several relations between different neutrosophic filters and neutrosophic topologies. Salama and Alagamy (2013) introduced the notion of filters on a neutrosophic set as a generalization of the notion of fuzzy filters. Recently, Hamidi et al. (2019) studied the notion of single-valued neutrosophic filters on EQ-algebras and its relationship with filters on these kind algebras. Öztürk and Jun (2018) presented neutrosophic ideals on BCK and BCI algebras with respect to neutrosophic points.

The present study is motivated by the work of Arockiarani and Antony Crispin Sweety (2016), in which they have considered the notions of lattice, ideal and filter in neutrosophic setting as single-valued neutrosophic sets on a given crisp lattice. More specifically, we deepen the study of these important notions by providing their various characterizations and properties. We pay particular attention to their characterizations based on the lattice min and max operations. Furthermore, the notion of prime single-valued neutrosophic ideal (respectively, filter) as interesting kinds is investigated.

This paper is organized as follows. In Section 2, we recall some basic concepts related to SVNSs and single-valued neutrosophic lattices (SVNLs). In Section 3, we provide interesting characterizations of single-valued neutrosophic ideals and filters based on the lattice min and max operations. In Section 4, we study the notion of prime single-valued neutrosophic ideals (respectively, filters) on a lattice, and show their interaction with some theoretical set-operations, as well as, with some associated single-valued neutrosophic sets. Finally, we present some concluding remarks in Section 5.

2. Basic concepts

In this section, we recall basic definitions and properties of SVNSs and some related notions that will be needed in the following sections. Throughout this paper, L always denotes a lattice $(L, \leqslant, \curlywedge, \curlyvee)$ and L^d its dual-order lattice $(L, \geqslant, \curlyvee, \curlywedge)$. Also, the notations $(\leq, \wedge, \vee)$ will used to refer the (usual order, $\min$, $\max$) on the real interval $[0, 1]$.

2.1. Single-valued neutrosophic sets

Smarandache (1998) introduced the notion of NSs as a generalization of Atanassov's intuitionistic fuzzy sets. For practical use of neutrosophic sets, Wang et al. (2010) proposed the notion of SVNSs as a subclass of NSs.

A SVNS A on a nonempty set X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle \mid x \in X\}$ which is characterized by a truth-membership function $\mathcal{T}_A : X \to [0,1]$, an indeterminacy-membership function $\mathcal{I}_A : X \to [0,1]$ and a falsity-membership function $\mathcal{F}_A : X \to [0,1]$.

Certainly, IFSs are SVNSs by setting $\mathcal{I}_A(x) = 1 - \mathcal{T}_A(x) - \mathcal{F}_A(x)$. The set of all SVNSs on a set X is denoted by $SVN(X)$.

In the setting of SVNSs, many set-operations are defined (see, e.g., Arockiarani et al. (2013); Saranya et al. (2020); Smarandach and Pramanik (2016); Wang et al. (2010); Yang et al. (2016)). The following are those needed in this paper.

Let A and B be SVNSs on a nonempty set X:
(i) $A \subseteq B$ if $\mathcal{T}_A(x) \leq \mathcal{T}_B(x),\ \mathcal{I}_A(x) \leq \mathcal{I}_B(x)$ and $\mathcal{F}_A(x) \geq \mathcal{F}_B(x),$ for any $x \in X$,
(ii) $A = B$ if $\mathcal{T}_A(x) = \mathcal{T}_B(x),\ \mathcal{I}_A(x) = \mathcal{I}_B(x)$ and $\mathcal{F}_A(x) = \mathcal{F}_B(x),$ for any $x \in X$,
(iii) $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle \mid x \in X\}$,
(iv) $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle \mid x \in X\}$,
(v) $\overline{A} = \{\langle x, \mathcal{F}_A(x), \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle \mid x \in X\}$,
(vi) $[A] = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), 1 - \mathcal{T}_A(x)\rangle \mid x \in X\}$,
(vii) $\langle A\rangle = \{\langle x, 1 - \mathcal{F}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle \mid x \in X\}$.

2.2. SVN-lattices, SVN-ideals and SVN-filters

The notion of single-valued neutrosophic lattice or fuzzy neutrosophic lattice as introduced by Arockiarani and Antony Crispin Sweety (Arockiarani and Antony Crispin Sweety (2016)) is a SVNS on a crisp lattice closed by their $\min$ and $\max$ operations.

A SVNS $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle \mid x \in L\}$ on a lattice L is called a single-valued neutrosophic lattice (SVN-lattice) if for any $a, b \in L$:
(i) $\mathcal{T}_A(a \curlyvee b) \geq \mathcal{T}_A(a) \wedge \mathcal{T}_A(b)$,
(ii) $\mathcal{T}_A(a \curlywedge b) \geq \mathcal{T}_A(a) \wedge \mathcal{T}_A(b)$,
(iii) $\mathcal{I}_A(a \curlyvee b) \geq \mathcal{I}_A(a) \wedge \mathcal{I}_A(b)$,
(iv) $\mathcal{I}_A(a \curlywedge b) \geq \mathcal{I}_A(a) \wedge \mathcal{I}_A(b)$,
(v) $\mathcal{F}_A(a \curlyvee b) \leq \mathcal{F}_A(a) \vee \mathcal{F}_A(b)$,
(vi) $\mathcal{F}_A(a \curlywedge b) \leq \mathcal{F}_A(a) \vee \mathcal{F}_A(b)$.

Example 2.1.

Let $L = \{0, a, b, 1\}$ be the lattice represented as in Figure 1. The SVNS $A = \{< 0, 0.5, 0.4, 0.1 >, < a, 0.4, 0.3, 0.5 >, < b, 0.4, 0.3, 0.3 >, < 1, 0.7, 0.6, 0.3 >\}$ on L is a SVN-lattice.

A SVNS $\mathbf{I} = \{\langle x, \mathcal{T}_I(x), \mathcal{I}_I(x), \mathcal{F}_I(x)\rangle \mid x \in L\}$ on a lattice L is called a single-valued neutrosophic ideal (SVN-ideal) if for all $a, b \in L$:
(i) $\mathcal{T}_I(a \curlyvee b) \geq \mathcal{T}_I(a) \wedge \mathcal{T}_I(b)$,
(ii) $\mathcal{T}_I(a \curlywedge b) \geq \mathcal{T}_I(a) \vee \mathcal{T}_I(b)$,

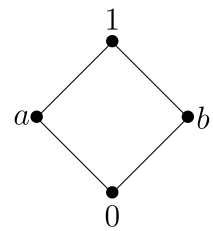

Figure 1. The lattice $(L, \leqslant, \curlywedge, \curlyvee)$ with $L = \{0, a, b, 1\}$.

(iii) $\mathcal{I}_I(a \curlyvee b) \geq \mathcal{I}_I(a) \wedge \mathcal{I}_I(b)$,
(iv) $\mathcal{I}_I(a \curlywedge b) \geq \mathcal{I}_I(a) \vee \mathcal{I}_I(b)$,
(v) $\mathcal{F}_I(a \curlyvee b) \leq \mathcal{F}_I(a) \vee \mathcal{F}_I(y)$,
(vi) $\mathcal{F}_I(a \curlywedge b) \leq \mathcal{F}_I(a) \wedge \mathcal{F}_I(b)$.

Dually, we introduce the notion of a SVN-filter on a lattice.

A SVNS $F = \{\langle x, \mathcal{T}_F(x), \mathcal{I}_F(x), \mathcal{F}_F(x)\rangle \mid x \in L\}$ on a lattice L is called a single-valued neutrosophic filter (SVN-filter) if for all $a, b \in L$:
(i) $\mathcal{T}_F(a \curlyvee b) \geq \mathcal{T}_F(a) \vee \mathcal{T}_F(b)$,
(ii) $\mathcal{T}_F(a \curlywedge b) \geq \mathcal{T}_F(a) \wedge \mathcal{T}_F(b)$,
(iii) $\mathcal{I}_F(a \curlyvee b) \geq \mathcal{I}_F(a) \vee \mathcal{I}_F(b)$,
(iv) $\mathcal{I}_F(a \curlywedge b) \geq \mathcal{I}_F(a) \wedge \mathcal{I}_F(b)$,
(v) $\mathcal{F}_F(a \curlyvee b) \leq \mathcal{F}_F(a) \wedge \mathcal{F}_F(b)$,
(vi) $\mathcal{F}_F(a \curlywedge b) \leq \mathcal{F}_F(a) \vee \mathcal{F}_F(b)$.

Certainly, IF-ideals (respectively, IF-filters) are SVN-ideals (respectively, SVN-filter).

Example 2.2.

Consider the lattice given in Example 2.1. Then
(i) the SVNS $I = \{< 0, 0.5, 0.6, 0.1 >, < a, 0.4, 0.5, 0.3 >, < b, 0.1, 0.3, 0.2 >, < 1, 0.1, 0.2, 0.3 >\}$ is a SVN-ideal on L,
(ii) the SVNS $F = \{< 0, 0.1, 0.2, 0.6 >, < a, 0.2, 0.3, 0.6 >, < b, 0.1, 0.2, 0.5 >, < 1, 0.4, 0.5, 0.3 >\}$ is a SVN-filter on L.

Remark 2.1.

As the crisp case, every SVN-ideal or SVN-filter on L is a SVN-lattice and not conversely. Indeed, Let $A = \{< 0, 0.3, 0.2, 0.1 >, < a, 0.4, 0.3, 0.5 >, < b, 0.4, 0.3, 0.3 >, < 1, 0.7, 0.6, 0.3 >\}$ be SVNS on the lattice L given in Example 2.1. It is clear that A is a SVN-lattice, but neither a SVN-ideal nor a SVN-filter on L.

3. Properties and characterizations of SVN-ideals and SVN-filters on a lattice

In this section, we investigate some properties and characterizations of the lattice ideals and filters in neutrosophic setting. We start with the following two results, in which the proofs are direct application of definitions.

Proposition 3.1.

Let $A \in SVN(L)$, then A is a SVN-ideal on L if and only if A is a SVN-filter on its dual-order lattice L^d.

Proof:

Suppose that A is a SVN-ideal on L and we show that A is a SVN-filter on its dual-order lattice L^d. We only show the first condition of SVN-filter, as the other conditions can be proved analogously. Let $x, y \in L$, $\mathcal{T}_F(x \curlyvee^d y) = \mathcal{T}_F(x \curlywedge y) \geq \mathcal{T}_F(x) \wedge \mathcal{T}_F(y) = \mathcal{T}_F(x) \vee^d \mathcal{T}_F(y)$. Similarly, we prove the converse implication. ■

Proposition 3.2.

The intersection of a family of SVN-ideals (respectively, SVN-filters) is also a SVN-ideal (respectively, SVN-filter).

Proof:

We only show the first condition of SVN-idedal, as all the other conditions of SVN-ideal (respectively, SVN-filter) can be proved analogously. Let $(A_i)_{i \in I}$ be a family of SVN-ideals on L and $x, y \in L$, then $\mathcal{T}_{\bigcap_{i \in I}(A_i)}(x \curlyvee y) = \min_{i \in I}(\mathcal{T}_{A_i}(x \curlyvee y) \geq \min_{i \in I}(\mathcal{T}_{A_i}(x) \wedge \mathcal{T}_{A_i}(y))$. Hence, $\mathcal{T}_{\bigcap_{i \in I}(A_i)}(x \curlyvee y) \geq \min_{i \in I}(\mathcal{T}_{A_i}(x)) \wedge \min_{i \in I}(\mathcal{T}_{A_i}(y)) = \mathcal{T}_{\bigcap_{i \in I}(A_i)}(x) \wedge \mathcal{T}_{\bigcap_{i \in I}(A_i)}(y)$. ■

The following result shows equivalences between conditions based on the lattice operations $(\curlywedge, \curlyvee)$ and conditions based on the lattice partial order $(\leqslant)$. Next, these equivalences will be used as tools to provide a characterization of SVN-ideals and SVN-filters on a lattice.

Proposition 3.3.

The following equivalences hold for any $A \in SVN(L)$ and $x, y \in L$:
(i) $(\mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)) \Leftrightarrow (x \leqslant y \Rightarrow \mathcal{T}_A(x) \geq \mathcal{T}_A(y))$,
(ii) $(\mathcal{T}_A(x \curlyvee y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)) \Leftrightarrow (x \leqslant y \Rightarrow \mathcal{T}_A(x) \leq \mathcal{T}_A(y))$,
(iii) $(\mathcal{I}_A(x \curlywedge y) \geq \mathcal{I}_A(x) \vee \mathcal{I}_A(y)) \Leftrightarrow (x \leqslant y \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_A(y))$,
(iv) $(\mathcal{I}_A(x \curlyvee y) \geq \mathcal{I}_A(x) \vee \mathcal{I}_A(y)) \Leftrightarrow (x \leqslant y \Rightarrow \mathcal{I}_A(x) \leq \mathcal{I}_A(y))$,
(v) $(\mathcal{F}_A(x \curlywedge y) \leq \mathcal{F}_A(x) \wedge \mathcal{F}_A(y)) \Leftrightarrow (x \leqslant y \Rightarrow \mathcal{F}_A(x) \leq \mathcal{F}_A(y))$,
(vi) $(\mathcal{F}_A(x \curlyvee y) \leq \mathcal{F}_A(x) \wedge \mathcal{F}_A(y)) \Leftrightarrow (x \leqslant y \Rightarrow \mathcal{F}_A(x) \geq \mathcal{F}_A(y))$.

Proof:

(i) Suppose that $\mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$, for any $x, y \in L$. If $x \leqslant y$, then $x \curlywedge y = x$. Since $\mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$, it follows that $\mathcal{T}_A(x) = \mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$. Hence, $\mathcal{T}_A(x) \geq \mathcal{T}_A(y)$.
Conversely, suppose that ($x \leqslant y \Rightarrow \mathcal{T}_A(x) \geq \mathcal{T}_A(y)$), for any $x, y \in L$. Then it follows that $\mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(x)$ and $\mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(y)$. Hence, $\mathcal{T}_A(x \curlywedge y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$.

(ii) Suppose that $\mathcal{T}_A(x \curlyvee y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$, for any $x, y \in L$. If $x \leqslant y$, then $x \curlyvee y = y$. Since $\mathcal{T}_A(x \curlyvee y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$, it follows that $\mathcal{T}_A(y) = \mathcal{T}_A(x \curlyvee y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$. Hence, $\mathcal{T}_A(x) \leq \mathcal{T}_A(y)$.
Conversely, let $x, y \in L$ such that ($x \leqslant y \Rightarrow \mathcal{T}_A(x) \leq \mathcal{T}_A(y)$). Then it follows that $\mathcal{T}_A(x) \leq \mathcal{T}_A(x \curlyvee y)$ and $\mathcal{T}_A(y) \leq \mathcal{T}_A(x \curlyvee y)$. Hence, $\mathcal{T}_A(x \curlyvee y) \geq \mathcal{T}_A(x) \vee \mathcal{T}_A(y)$.

Similarly, we prove (iii) and (v) as (i). Also, (iv) and (vi) as (ii). ■

The following corollaries present several properties of SVN-ideals and SVN-filters on a given lattice.

Corollary 3.1.

Let I be a SVN-ideal on L, then it holds that
(i) $\mathcal{T}_I : L \to [0,1]$ is an antitone mapping, (i.e., If $x \leqslant y$, then $\mathcal{T}_I(x) \geq \mathcal{T}_I(y)$, for any $x, y \in L$),
(ii) $\mathcal{I}_I : L \to [0,1]$ is an antitone mapping,
(iii) $\mathcal{F}_I : L \to [0,1]$ is a monotone mapping, (i.e., If $x \leqslant y$, then $\mathcal{F}_I(x) \leq \mathcal{F}_I(y)$, for any $x, y \in L$).

Proof:

We only show (i), as (ii) and (iii) can be proved analogously. Suppose that I is a SVN-ideal on L, then it holds that $\mathcal{T}_I(a \curlywedge b) \geq \mathcal{T}_I(a) \vee \mathcal{T}_I(b)$. Now, Proposition 3.3 (i) guarantees that $\mathcal{T}_I : L \to [0,1]$ is an antitone mapping. ■

Corollary 3.2.

For any SVN-filter F on L, it holds that $\mathcal{T}_F$, $\mathcal{I}_F$ are monotone mappings and $\mathcal{F}_F$ is an antitone mapping.

Proof:

Follows by combining Proposition 3.1 and Corollary 3.1. ■

Corollary 3.3.

If L has smallest element $\perp$ and greatest element $\top$, then any SVN-ideal I on L satisfying:
(i) $\mathcal{T}_I(\perp) = \max \mathcal{T}_I(L)$ and $\mathcal{T}_I(\top) = \min \mathcal{T}_I(L)$, where $\mathcal{T}_I(L) = \{\mathcal{T}_I(x) \mid x \in L\}$,
(ii) $\mathcal{I}_I(\perp) = \max \mathcal{I}_I(L)$ and $\mathcal{I}_I(\top) = \min \mathcal{I}_I(L)$, where $\mathcal{I}_I(L) = \{\mathcal{I}_I(x) \mid x \in L\}$,
(iii) $\mathcal{F}_I(\perp) = \min \mathcal{F}_I(L)$ and $\mathcal{F}_I(\top) = \max \mathcal{F}_I(L)$, where $\mathcal{F}_I(L) = \{\mathcal{F}_I(x) \mid x \in L\}$.

Proof:

We only show (i), as (ii) and (iii) can be proved analogously. Suppose that I is a SVN-ideal on L, then it holds from Corollary 3.1 that $\mathcal{T}_I : L \to [0,1]$ is an antitone mapping. Thus, $\mathcal{T}_I(\bot) = \max \mathcal{T}_I(L)$ and $\mathcal{T}_I(\top) = \min \mathcal{T}_I(L)$. ■

Corollary 3.4.

If L has smallest element $\bot$ and greatest element $\top$, then any SVN-filter F on L satisfying :
(i) $\mathcal{T}_F(\bot) = \min \mathcal{T}_I(L)$ and $\mathcal{T}_F(\top) = \max \mathcal{T}_F(L)$, where $\mathcal{T}_F(L) = \{\mathcal{T}_F(x) \mid x \in L\}$,
(ii) $\mathcal{I}_F(\bot) = \min \mathcal{I}_I(L)$ and $\mathcal{I}_F(\top) = \max \mathcal{I}_F(L)$, where $\mathcal{I}_F(L) = \{\mathcal{I}_F(x) \mid x \in L\}$,
(iii) $\mathcal{F}_F(\bot) = \max \mathcal{F}_I(L)$ and $\mathcal{F}_F(\top) = \min \mathcal{F}_F(L)$, where $\mathcal{F}_F(L) = \{\mathcal{F}_F(x) \mid x \in L\}$.

Proof:

Follows by combining Proposition 3.1 and Corollary 3.3. ■

The following two theorems provide characterizations of SVN-ideal (respectively, SVN-filter) on a lattice.

Theorem 3.1.

I is a SVN-ideal on L if and only if for any $x, y \in L$, the following conditions are satisfied:
(i) $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$,
(ii) $\mathcal{I}_I(x \curlyvee y) = \mathcal{I}_I(x) \wedge \mathcal{I}_I(y)$,
(iii) $\mathcal{F}_I(x \curlyvee y) = \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$.

Proof:

Suppose that I is a SVN-ideal on L and we show that the above three conditions are satisfied. By hypothesis we have that $\mathcal{T}_I(x \curlyvee y) \geq \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, $\mathcal{I}_I(x \curlyvee y) \geq \mathcal{I}_I(x) \wedge \mathcal{I}_I(y)$ and $\mathcal{F}_I(x \curlyvee y) \leq \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$. Further, for any $x, y \in L$ it holds from Corollary 3.1 that $(\mathcal{T}_I(x) \geq \mathcal{T}_I(x \curlyvee y)$, $\mathcal{I}_I(x) \geq \mathcal{I}_I(x \curlyvee y)$, $\mathcal{F}_I(x) \leq \mathcal{F}_I(x \curlyvee y))$ and $(\mathcal{T}_I(y) \geq \mathcal{T}_I(x \curlyvee y)$, $\mathcal{I}_I(y) \geq \mathcal{I}_I(x \curlyvee y)$, $\mathcal{F}_I(y) \leq \mathcal{F}_I(x \curlyvee y))$. Hence, $\mathcal{T}_I(x) \wedge \mathcal{T}_I(y) \geq \mathcal{T}_I(x \curlyvee y)$, $\mathcal{I}_I(x) \wedge \mathcal{I}_I(y) \geq \mathcal{I}_I(x \curlyvee y)$ and $\mathcal{F}_I(x) \vee \mathcal{F}_I(y) \leq \mathcal{F}_I(x \curlyvee y)$. Thus, $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, $\mathcal{I}_I(x \curlyvee y) = \mathcal{I}_I(x) \wedge \mathcal{I}_I(y)$ and $\mathcal{F}_I(x \curlyvee y) = \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$.

Conversely, suppose that $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, $\mathcal{I}_I(x \curlyvee y) = \mathcal{I}_I(x) \wedge \mathcal{I}_I(y)$ and $\mathcal{F}_I(x \curlyvee y) = \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$, for any $x, y \in L$. It is obvious to see for any $x, y \in L$ that $\mathcal{T}_I(x \curlyvee y) \geq \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, $\mathcal{I}_I(x \curlyvee y) \geq \mathcal{I}_I(x) \wedge \mathcal{I}_I(y)$ and $\mathcal{F}_I(x \curlyvee y) \leq \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$. Next, we will show that $\mathcal{T}_I(x \curlywedge y) \geq \mathcal{T}_I(x) \vee \mathcal{T}_I(y)$, $\mathcal{I}_I(x \curlywedge y) \geq \mathcal{I}_I(x) \vee \mathcal{I}_I(y)$ and $\mathcal{F}_I(x \curlywedge y) \leq \mathcal{F}_I(x) \wedge \mathcal{F}_I(y)$, for any $x, y \in L$. Let $x, y \in L$, since $x \curlyvee (x \curlywedge y) = x$ and $y \curlyvee (x \curlywedge y) = y$, then it holds that $\mathcal{T}_I(x \curlyvee (x \curlywedge y)) = \mathcal{T}_I(x)$ and $\mathcal{T}_I(y \curlyvee (x \curlywedge y)) = \mathcal{T}_I(y)$. From hypothesis (i) and (ii), it follows that $\mathcal{T}_I(x) \wedge \mathcal{T}_I(x \curlywedge y) = \mathcal{T}_I(x)$ and $\mathcal{T}_I(y) \wedge \mathcal{T}_I(x \curlywedge y) = \mathcal{T}_I(y)$. Hence, $\mathcal{T}_I(x \curlywedge y) \geq \mathcal{T}_I(x)$ and $\mathcal{T}_I(x \curlywedge y) \geq \mathcal{T}_I(y)$. Thus, $\mathcal{T}_I(x \curlywedge y) \geq \mathcal{T}_I(x) \vee \mathcal{T}_I(y)$, for any $x, y \in L$. In the same way, we obtain that $\mathcal{I}_I(x \curlywedge y) \geq \mathcal{I}_I(x) \vee \mathcal{I}_I(y)$ and $\mathcal{F}_I(x \curlywedge y) \leq \mathcal{F}_I(x) \wedge \mathcal{F}_I(y)$, for any $x, y \in L$. Therefore, I

is a SVN-ideal on L. ■

Theorem 3.2.

F is a SVN-filter on L if and only if for any $x, y \in L$, the following conditions are satisfied:
(i) $\mathcal{T}_F(x \curlywedge y) = \mathcal{T}_F(x) \wedge \mathcal{T}_F(y)$,
(ii) $\mathcal{I}_F(x \curlywedge y) = \mathcal{I}_F(x) \wedge \mathcal{I}_F(y)$,
(ii) $\mathcal{F}_F(x \curlywedge y) = \mathcal{F}_F(x) \vee \mathcal{F}_F(y)$.

Proof:

The proof can be obtained by direct application of Proposition 3.1 and Theorem 3.1. ■

The following corollaries characterize crisp (fuzzy) ideals and intuitionstic fuzzy ideals (respectively, crisp (fuzzy) filters and intuitionstic fuzzy filters) on a given lattice.

Corollary 3.5.

For any crisp (fuzzy) sets I and F on L, the following equivalences hold:
(i) I is a crisp (fuzzy) ideal on L if and only if $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, for any $x, y \in L$,
(ii) F is a crisp (fuzzy) filter on L if and only if $\mathcal{T}_F(x \curlywedge y) = \mathcal{T}_F(x) \wedge \mathcal{T}_F(y)$, for any $x, y \in L$.

Proof:

We only show (i), as (ii) can be proved by using Proposition 3.1 and (i). Since any crisp (fuzzy) ideal is a SVN-ideal on L by setting that $\mathcal{I}_A(x) = 0$ and $\mathcal{F}_A(x) = 1 - \mathcal{T}_A(x)$, it follows from Theorem 3.1 that I is a crisp (fuzzy) ideal on L if and only if $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, for any $x, y \in L$. ■

Corollary 3.6.

For any IFSs I and F on L, the following equivalences hold:
(i) I is an IF-ideal on L if and only if for any $x, y \in L$, the following two conditions are satisfied:
(a) $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, (b) $\mathcal{F}_I(x \curlyvee y) = \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$.
(ii) F is an IF-filter on L if and only if for any $x, y \in L$, the following two conditions are satisfied:
(a) $\mathcal{T}_F(x \curlywedge y) = \mathcal{T}_F(x) \wedge \mathcal{T}_F(y)$, (b) $\mathcal{F}_F(x \curlywedge y) = \mathcal{F}_F(x) \vee \mathcal{F}_F(y)$.

Proof:

We only show (i), as (ii) can be proved by using Proposition 3.1 and (i). Since any IF-ideal is a SVN-ideal on L by setting that $\mathcal{I}_A(x) = 1 - \mathcal{T}_A(x) - \mathcal{F}_A(x)$, it follows from Theorem 3.1 that I is an IF-ideal on L if and only if for any $x, y \in L$, the following two conditions are satisfied: (a) $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$, (b) $\mathcal{F}_I(x \curlyvee y) = \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$. ■

4. Prime SVN-ideals and prime SVN-filters on a lattice

This section is devoted to study the notion of prime SVN-ideals (respectively, SVN-filters) on a given lattice.

4.1. Definitions and basic characterization

Given a SVN-ideal I (respectively, SVN-filter F) on a lattice L.

I is called a prime SVN-ideal if, for any $x, y \in L$, it holds that
(i) $\mathcal{T}_I(x \curlywedge y) \leq \mathcal{T}_I(x) \vee \mathcal{T}_I(y)$,
(ii) $\mathcal{I}_I(x \curlywedge y) \leq \mathcal{I}_I(x) \vee \mathcal{I}_I(y)$,
(iii) $\mathcal{F}_I(x \curlywedge y) \geq \mathcal{F}_I(x) \wedge \mathcal{F}_I(y)$.

Dually, F is called a prime SVN-filter if, for any $x, y \in L$, it holds that
(i) $\mathcal{T}_F(x \curlyvee y) \leq \mathcal{T}_F(x) \vee \mathcal{T}_F(y)$,
(ii) $\mathcal{I}_F(x \curlyvee y) \leq \mathcal{I}_F(x) \vee \mathcal{I}_F(y)$,
(iii) $\mathcal{F}_F(x \curlyvee y) \geq \mathcal{F}_F(x) \wedge \mathcal{F}_F(y)$.

The above Theorem 3.1 and Proposition 3.3 lead to the following result which characterize prime single-valued neutrosophic ideals.

Proposition 4.1.

I is a prime SVN-ideal on L if and only if for any $x, y \in L$, the following conditions hold:
(i) $\mathcal{T}_I(x \curlyvee y) = \mathcal{T}_I(x) \wedge \mathcal{T}_I(y)$,
(ii) $\mathcal{T}_I(x \curlywedge y) = \mathcal{T}_I(x) \vee \mathcal{T}_I(y)$,
(iii) $\mathcal{I}_I(x \curlyvee y) = \mathcal{I}_I(x) \wedge \mathcal{I}_I(y)$,
(iv) $\mathcal{I}_I(x \curlywedge y) = \mathcal{I}_I(x) \vee \mathcal{I}_I(y)$,
(v) $\mathcal{F}_I(x \curlyvee y) = \mathcal{F}_I(x) \vee \mathcal{F}_I(y)$,
(vi) $\mathcal{F}_I(x \curlywedge y) = \mathcal{F}_I(x) \wedge \mathcal{F}_I(y)$.

Proof:

The conditions (i), (iii) and (v) are obtained directly from Theorem 3.1. The other cases (ii), (iv) and (vi) can be deduced by using the above definition of prime SVN-ideal and Proposition 3.3. ■

Example 4.1.

Let L be the lattice given by the Hasse diagram in Figure 1. Then $I = \{< 0, 0.5, 0.4, 0.1 >, < a, 0.4, 0.3, 0.2 >, < b, 0.3, 0.2, 0.1 >, < 1, 0.3, 0.2, 0.5 >\}$ is a prime SVN-ideal on L.

Similarly, Theorem 3.2 and Proposition 3.3 lead to the following result which characterize prime SVN-filters.

Proposition 4.2.

F is a prime SVN-filter on L if and only if for any $x, y \in L$, the following conditions hold:
(i) $\mathcal{T}_F(x \curlyvee y) = \mathcal{T}_F(x) \vee \mathcal{T}_F(y)$,
(ii) $\mathcal{T}_F(x \curlywedge y) = \mathcal{T}_F(x) \wedge \mathcal{T}_F(y)$,
(iii) $\mathcal{I}_F(x \curlyvee y) = \mathcal{I}_F(x) \vee \mathcal{I}_F(y)$,
(iv) $\mathcal{I}_F(x \curlywedge y) = \mathcal{I}_F(x) \wedge \mathcal{I}_F(y)$,
(v) $\mathcal{F}_F(x \curlyvee y) = \mathcal{F}_F(x) \wedge \mathcal{F}_F(y)$,
(vi) $\mathcal{F}_F(x \curlywedge y) = \mathcal{F}_F(x) \vee \mathcal{F}_F(y)$.

Proof:

The proof can be obtained by direct application of Proposition 3.1 and the previous Proposition 4.1. ■

4.2. Set-operations on prime SVN-ideals (respectively, prime SVN-filters)

In this subsection, we discuss some set-operations on prime SVN-ideals (respectively, prime SVN-filters).

Proposition 4.3.

Let $(A_i)_{i \in I}$ be a family of SVNSs on L. Then
(i) If A_i is a prime SVN-ideal on L, for any $i \in I$, then $\bigcap\limits_{i \in I} A_i$ is a prime SVN-ideal on L,
(ii) If A_i is a prime SVN-filter on L, for any $i \in I$, then $\bigcap\limits_{i \in I} A_i$ is a prime SVN-filter on L.

Proof:

We only give the proof of (i), as (ii) can be proved analogously by using Proposition 3.1.

Suppose that A_i is a prime SVN-ideal on L, for any $i \in I$. From Proposition 3.2, it follows that $\bigcap\limits_{i \in I} A_i$ is a SVN-ideal on L. It remains to show that $\bigcap\limits_{i \in I} A_i$ is prime. Let $x, y \in L$ such that $x \curlywedge y \in \bigcap\limits_{i \in I} A_i$. Then, it follows that $x \curlywedge y \in A_i$, for any $i \in I$. Since for any $i \in I$, A_i is a prime SVN-ideal, it follows that $\mathcal{T}_{A_i}(x \curlywedge y) \leq \mathcal{T}_{A_i}(x) \vee \mathcal{T}_{A_i}(y)$, $\mathcal{I}_{A_i}(x \curlywedge y) \leq \mathcal{I}_{A_i}(x) \vee \mathcal{I}_{A_i}(y)$ and $\mathcal{F}_{A_i}(x \curlywedge y) \geq \mathcal{F}_{A_i}(x) \wedge \mathcal{F}_I(y)$, for any $i \in I$. This implies that $\mathcal{T}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \leq \mathcal{T}_{A_i}(x \curlywedge y) \leq \mathcal{T}_{A_i}(x) \vee \mathcal{T}_{A_i}(y)$, $\mathcal{I}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \leq \mathcal{I}_{A_i}(x \curlywedge y) \leq \mathcal{I}_{A_i}(x) \vee \mathcal{I}_{A_i}(y)$ and $\mathcal{F}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \geq \mathcal{F}_{A_i}(x \curlywedge y) \geq \mathcal{F}_{A_i}(x) \wedge \mathcal{F}_I(y)$, for any $i \in I$. Hence, $\mathcal{T}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \leq \bigwedge_{i \in I}(\mathcal{T}_{A_i}(x) \vee \mathcal{T}_{A_i}(y))$, $\mathcal{I}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \leq \bigwedge_{i \in I}(\mathcal{I}_{A_i}(x) \vee \mathcal{I}_{A_i}(y))$ and $\mathcal{F}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \geq \bigvee_{i \in I}(\mathcal{F}_{A_i}(x) \wedge \mathcal{F}_I(y))$. Thus, $\mathcal{T}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \leq \mathcal{T}_{\bigcap\limits_{i \in I} A_i}(x) \vee \mathcal{T}_{\bigcap\limits_{i \in I} A_i}(y)$, $\mathcal{I}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \leq \mathcal{I}_{\bigcap\limits_{i \in I} A_i}(x) \vee \mathcal{I}_{\bigcap\limits_{i \in I} A_i}(y)$ and $\mathcal{F}_{\bigcap\limits_{i \in I} A_i}(x \curlywedge y) \geq \mathcal{F}_{\bigcap\limits_{i \in I} A_i}(x) \wedge \mathcal{F}_{\bigcap\limits_{i \in I} A_i}(y)$. Therefore, $\bigcap\limits_{i \in I} A_i$ is a prime SVN-ideal on L. ■

The following proposition discusses the relationship between a SVN-ideal (respectively, SVN-filter) and its complement.

Proposition 4.4.

For any $A \in SVN(L)$, the following equivalences hold:
(i) A is a prime SVN-ideal if and only if $\overline{A}$ is a prime SVN-filter on L,
(ii) A is a prime SVN-filter if and only if $\overline{A}$ is a prime SVN-ideal on L.

Proof:

(i) Suppose that A is a prime SVN-ideal, for any $x, y \in L$ it follows from Proposition 4.1 that

$$\mathcal{T}_{\overline{A}}(x \curlyvee y) = \mathcal{F}_A(x \curlyvee y) = \mathcal{F}_A(x) \vee \mathcal{F}_A(y) = \mathcal{T}_{\overline{A}}(x) \vee \mathcal{T}_{\overline{A}}(y),$$

and

$$\mathcal{T}_{\overline{A}}(x \curlywedge y) = \mathcal{F}_A(x \curlywedge y) = \mathcal{F}_A(x) \wedge \mathcal{F}_A(y) = \mathcal{T}_{\overline{A}}(x) \wedge \mathcal{T}_{\overline{A}}(y)\,.$$

In a similar way, we prove that $\mathcal{I}_{\overline{A}}(x \curlyvee y) = \mathcal{I}_{\overline{A}}(x) \vee \mathcal{I}_{\overline{A}}(y)$, $\mathcal{I}_{\overline{A}}(x \curlywedge y) = \mathcal{I}_{\overline{A}}(x) \wedge \mathcal{I}_{\overline{A}}(y)$, $\mathcal{F}_{\overline{A}}(x \curlyvee y) = \mathcal{F}_{\overline{A}}(x) \wedge \mathcal{F}_{\overline{A}}(y)$ and $\mathcal{F}_{\overline{A}}(x \curlywedge y) = \mathcal{F}_{\overline{A}}(x) \vee \mathcal{F}_{\overline{A}}(y)$. Applying Proposition 4.2 guarantees that $\overline{A}$ is a prime SVN-filter on L. The converse follows from Proposition 3.1 and the first implication.

(ii) Follows from the fact that $A = \overline{\overline{A}}$ and (i). ∎

Proposition 4.5.

Let $A \in SVN(L)$, then A is a prime SVN-ideal (respectively, prime SVN-filter) if and only if $[A]$ is a prime SVN-ideal (respectively, prime SVN-filter) on L.

Proof:

We only give the proof of the case of ideal, as the case of filter can be proved analogously by using Proposition 3.1. Suppose that A is a prime SVN-ideal on a lattice L. It is clear that $[A] = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), 1 - \mathcal{T}_A(x)\rangle \mid x \in X\}$ is a SVN-ideal on L. Next, we show that $[A]$ is prime. We have that

$$\mathcal{T}_{[A]}(x \curlywedge y) = \mathcal{T}_A(x \curlywedge y) = \mathcal{T}_A(x) \vee \mathcal{T}_A(y) = \mathcal{T}_{[A]}(x) \vee \mathcal{T}_{[A]}(y),$$

and

$$\mathcal{I}_{[A]}(x \curlywedge y) = \mathcal{I}_A(x \curlywedge y) = \mathcal{I}_A(x) \vee \mathcal{I}_A(y) = \mathcal{I}_{[A]}(x) \vee \mathcal{I}_{[A]}(y)\,.$$

Also,

$$\mathcal{F}_{[A]}(x \curlywedge y) = 1 - \mathcal{T}_A(x \curlywedge y) = 1 - (\mathcal{T}_A(x) \vee \mathcal{T}_A(y)) = (1 - \mathcal{T}_A(x)) \wedge (1 - \mathcal{T}_A(y)) = \mathcal{F}_{[A]}(x) \wedge \mathcal{F}_{[A]}(y)\,.$$

We conclude that $[A]$ is a prime SVN-ideal on L. Conversely, suppose that $[A]$ is a prime SVN-ideal. By using the same steps we get that A is a prime SVN-ideal on L. ∎

Proposition 4.6.

Let $A \in SVN(L)$, then A is a prime SVN-ideal (respectively, prime SVN-filter) if and only if $\langle A\rangle$ is a prime SVN-ideal (respectively, prime SVN-filter) on L.

Proof:

The proof is analogous to that of Proposition 4.5 by using the definition of $\langle A\rangle$ instead of $[A]$. ∎

Remark 4.1.

The advantages of the presented work are multiple. The biggest advantage of these characterizations is that they facilitate the study and the representations of SVN-ideal (respectively, SVN-filter) on a given lattice. On the other hand, these results allow us to improve the study of some types of SVN-ideal (respectively, SVN-filter).

5. Conclusion and Future Work

In this article, we have studied properties of SVN-ideals and SVN-filters on a lattice, and provided their various characterizations. We have introduced and studied the notions of prime SVN-ideal and prime SVN-filter on a lattice as interesting kinds, and we have discussed their set-operations, complement and some of their associated sets. We anticipate that these notions of SVN-ideals (respectively, SVN-filters) will facilitate the study and the representations of the different kinds of SVN-lattices. Due to the usefulness of these notions, we think it makes sense to study these notions for other types of lattices. Future efforts will be directed to the type of lattices with respect to SVN-order relations.

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